A METHOD TO ASSESS THE GEOMETRIC PURITY OF SHADOW LINES: A CASE STUDY OF DAYLIGHTING CONDITIONS

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Abstract

Some naturally occurring proportional relationships and geometries have been associated with harmony and the perception of beauty. The Golden Ratio, for example has been a source of inspiration and composition for many art specialties, including architecture. Although there is a relatively large body of literature demonstrating that certain geometries, those with mathematically pure expressions, are visually pleasing, little scholarship exists concerning how these relationships might be integrated into architectural design. This paper aims to describe a method that allows architects to test the geometric purity of lines formed by the edge of light and shadow that result from proposed daylight conditions. The method was applied to different daylighting conditions where the geometric relationships between the aperture, the room walls, and the sun position create the resulting shadow lines. The test for purity includes a statistical comparison, using R^2 as an indicator of correlation between the resulting shadow lines and similar lines with a pure mathematical expression. This methodology is an initial step for developing a design decision-making representation tool that considers integrating geometric purity of daylight within the architectural design process.

Introduction

Philosophers, scientists, and mathematicians have found an association between the geometry of viewed objects and the perception of beauty; geometries with mathematically pure expressions seem to please our brains. Investigations into many natural objects, such as seashells, animal horns, and sunflowers, show that a proportional relationship, the Fibonacci series, governs their form (Cook, 1979; Ghyka, 2014; Huntley, 2012). Furthermore, such pure geometries are naturally occurring and fairly common, and are not governed by rules such as social class and cultural background (Scruton, 2011). There is a body of literature incorporating geometric purity into different disciplines, such as auto design (Leder & Carbon, 2005), furniture (Dazkir & Read, 2012), and everyday objects (Bar & Neta, 2006). Other research suggests that people typically find curved lines and objects more pleasing than angular and straight shapes (Silvia & Barona, 2008). Unfortunately, there is a gap in the research concerning how to incorporate these aesthetically pleasing geometries into indoor environments.

"Architecture is essentially an extension of nature into the man-made realm, providing the ground for perception and the horizon of experiencing and understanding the world." (Pallasmaa, 2012, p. 44).

Architects have long relied on mathematics to establish a visual framework for their buildings. Many architects and architectural researchers contend that perception is sensitive to the mathematical relationships of design elements (Kellert, 2012; Lidwell et al., 2003). In his book "Towards a New Architecture," Le Corbusier (2013) argued that the problem with modern architecture lies in its lack of mathematical principles. Therefore, he suggests that architectural elements, such as form and plan, must be studied mathematically. This might suggest the need for a representation tool that considers the integration of geometrically pure geometries and lines during the early stages of the design process. To do this, a method must first be found to evaluate the purity of the lines that result from the proposed design. In this research, this method was developed and applied to measure the degree of purity for shadow lines from daylighting conditions that resulted from variable aperture, room wall configuration, and position of the sun as it shines through the aperture.

As a spatial art, architecture often uses daylight as an element to affect the visual-spatial experience; as a result, architects have long capitalized on the qualitative and quantitative benefits of daylight (Holl et al., 1994; Rasmussen, 1964; Millet & Barrett, 1996; Zumthor, 2006). Steven Holl's (2000) exploration of light, as presented in "The Speed of Shadow" in "Parallax," presents an opportunity to apply the natural curves of daylight, pure geometries, into spatial conditions. Holl proposes an architectural language through an exploration of how curved walls and differently shaped roof openings can yield varied curves of light and shadow. Building on Holl's exploration, the present work describes the simulation and evaluation of geometric purity for shadow patterns for three daylighting conditions, as shown in Figure 1.

Methodology

The purpose of this paper was to develop a tool that allows architects to assess the geometric purity of lines in the visual field. In applying this tool, the researcher used two software programs: Autodesk AutoCAD 2018 and CurveExpert Professional 2.6.5. The suggested method incorporates four main phases: drawing the line, extracting the coordinates, testing degree of purity, and finally determining the R^2 value. This method was applied to three different daylight patterns that resulted from different spatial conditions. The light and shadow patterns were tested under sunlight for different times of the year.

SIMULATION VARIABLES

Daylighting conditions were extracted from different spatial conditions and different sun angles. All models placed in the latitude of Blacksburg, VA, (37.2296° N, 80.4139° W). Two conditions were used in constructing the northern wall and the vertical opening: straight line and semicircle. The first model consists of a linear northern wall and a semicircular vertical opening. A sample of daylight condition was taken at sun altitude of 55.23° and azimuth of 172.71°. The second model consists of a semicircular northern wall and a linear vertical opening. A sample of daylight condition was taken at sun altitude of 63.69° and azimuth of 199.68°. The third model consists of a semicircular northern wall and vertical opening. A sample of daylight condition was taken at sun altitude of 51.23° and azimuth of 148.44°. The spatial conditions can be seen in Figure 2. All three models were tested under clear sky conditions which cause harsh shadow patterns.

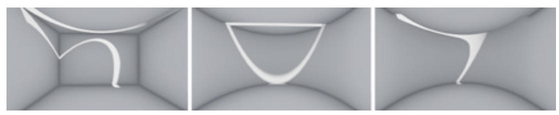

Figure 1: The three interior views of resulting daylighting conditions (conditions one to three are shown from left to right). (Source: Authors.)

Figure 2: Top view of the three models (model one to three are shown from left to right). (Source: Authors.)

COORDINATES SETUP

The first step in the assessment process is to place the image of the room on a two-dimensional grid so that the X-axis meets the bottom of the image and the Y-axis meets the left part of the image, corresponding to (0,0). Next, the polyline command is used by clicking on the starting point of the line to be evaluated, such as the line between light and shadow. Next the length command is applied by typing L. The line to be evaluated can now be drawn via several polygons. Upon reaching the last point along the selected line, press the spacebar to end the command polygon. Then press the drawn line with the left button and write the command pedit, then choose the command fit. This step will convert the drawn line from the polygons to a smooth curve, thus completing the first step. A flow chart illustrating the steps in Figure 3.

LINE DRAWING

Next, select the divide command and click on the curve, while choosing the number of points (coordinates) you want to export. For example, press the number 6 and then press the spacebar. At this stage you could also add the point manually by using the command point and placing points on the curve. Then select the points on the curve and choose the list command. The coordinates of each point will appear as X and Y at the bottom of the bar. Note: The starting and end points must be on the X-axis. This step can be done by applying the command align.

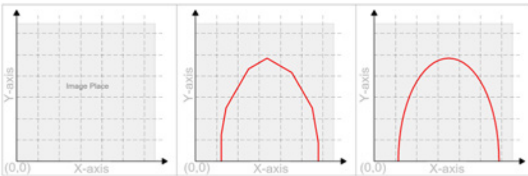

Figure 3: Flow chart illustrating coordinates setup (shown from left to right). (Source: Authors.)

TEST FOR GEOMETRIC PURITY

Run the Curve Expert program and write the extracted coordinates of the X and Y points into the table. Click on calculate the nonlinear regression, then place a check mark on the nonlinear model and click OK. In this way, the program will apply all the equations to the data. Then a list of all equations will be sorted by the highest value of R^2 to the lowest. These equations express the types of curves that have a mathematical purity such as quadratic and parabola.

R-SQUARED INDICATION

The R^2 coefficient is a statistical measure that shows how closely real data converges to the fitted line. The closer the R^2 value to 1.00, the more regression predictions match the data. In the research context, the R^2 indicated how curvature of shadow lines corresponds to the given equation in Curve Expert. The higher value of R^2, the shadow line belongs to the pure mathematical equation. Precisely, a threshold of 0.95 was chosen to determine the mathematical purity. To illustrate, if the value of R^2 is less than 0.95 for the fitted line, the geometry is not considered pure.

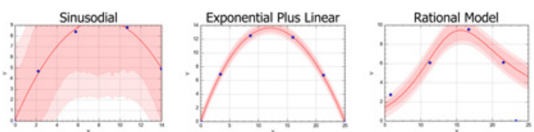

Figure 4: Results for daylighting conditions one to three (shown from left to right). (Source: Authors.)

Results

After testing three daylighting conditions, the results showed that all shadow lines were mathematically pure as shown graphically in Figure 4. The determinants of geometric purity depend on the R^2 value and the mathematical expression. The first condition shows a sinusoidal equation with R^2 of 0.99 that can be represented as:

$$y = a + b \cos (cx + d)$$

The second condition shows an exponential plus linear function with R^2 of 0.99 that can be represented as:

$$y = a + br^x + c$$

The third condition shows a rational function with R^2 of 0.98 that can be represented as:

$$y = a + bx / 1 + cx + dx^2$$

Conclusion

The paper touched on the importance of pure geometrical characteristics and their visual relationship to our sense of beauty, choosing daylight as an example. The literature shows that lines with mathematically pure expressions are visually pleasing. A mathematical method for quantifying the curvature distribution was proposed based on AutoCAD software and Curve Expert. The method lies on using R^2 as an indicator of linking between the resulting shadow lines and similar lines with a pure mathematical expression. The tool was tested on three models consisting of two spatial conditions: linear and semi-circular. Each model used the two conditions interchangeably between a wall and a vertical aperture. Models were tested under clear sky and different sun positions. Results from the three experiments showed that the shadow lines were mathematically pure.

Future Research

The next step is to use the tool for building digital models that have pure and impure spatial conditions that includes walls, apertures, and daylighting conditions. Future research should compare quantitative measures with subjective evaluations. The findings will validate the researcher to develop a design decision-making representation tool that easily integrates the suggested method into early stages of the architectural design process.

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